APPROACHES TO TEACHING MATHEMATICS:
Mapping the Domains of Knowledge, Skills, and Dispositions

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> *Teacher education has suffered too long from too many answers and too few questions... it may be time to worry less about finding the right answers and more about asking the right questions.* (Silberman, 1970, p. 470)

The Center for Research on Teacher Education will be investigating how teacher education prepares teachers to teach mathematics. What are the important questions to ask? What should we look at? What should we try to find out about? These decisions require a sense of what it might mean to prepare to teach mathematics: How should mathematics teachers be prepared? What do teachers need to know--about mathematics, about learners and learning, about curriculum? What should they be able to do? Are there crucial dispositions required for good mathematics teaching?

Answers to these central questions depend of course on one's view of what counts as "good mathematics teaching." The purpose of this memo is to provide a set of perspectives on what it means to teach mathematics well, in order to help us focus on dimensions that may play a key part in the preparation of beginning mathematics teachers and the continuing education of experienced teachers. Drawing from a review of literature in a number of related fields, we present and characterize four dominant approaches that seem commonly advocated. We begin with an overview of what we mean by an "approach to teaching mathematics," describing a set of factors that interact to shape a particular approach. Then, for each approach, we sketch its dominant focus and form, as well as what the teacher who teaches in this way needs to know, be able to do, and be like.

Approaches to Teaching Mathematics

Researchers have explored a number of alternative approaches to mathematics teaching (see, for example, Carpenter, Fennema, & Peterson, 1984; Steinberg, Haymore, & Marks, 1985; Stephens & Romberg, 1985; Hansen, McCann, & Myers, 1985; Myers, Hansen, Robson, & McCann, 1983). Various labels are used to characterize these different approaches--for example: "cognitively-guided instruction," "conceptual change teaching," "rote teaching strategies," "rule-based teaching," and "mastery learning." A review of the literature in teacher education, mathematics education, philosophy of mathematics, philosophy of education, and research on teaching and learning suggests that there are at least four dominant and distinctive views of how mathematics should be taught:

1. Learner-focused: mathematics teaching that focuses on the learner's personal construction of mathematical knowledge;

2. Content-focused with an emphasis on conceptual understanding: mathematics teaching that is driven by the content itself but emphasizes conceptual understanding;

3. Content-focused with an emphasis on performance: mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures; and

4. Classroom-focused: mathematics teaching based on research knowledge about effective classrooms.

These views, or teaching approaches, appear to be influenced partly by different conceptions of mathematics as a discipline and as a school subject. They also reflect different theories of learning and beliefs about effective teaching, as well as implicit assumptions about the purposes of schools and school learning.

The following question guided our deliberation about the characteristics of each teaching approach, "If a person who advocates Approach X were to observe someone else teaching mathematics, what would they pay most attention to?" In other words, what would they be most likely to notice and critique? While, in each teaching approach, there is a concern for the essential elements of classroom instruction--the learner, the content of instruction, and the classroom strategies used during lessons--the approaches are distinguished by the factor that governs the focus of classroom activity. It is a figure-ground distinction. The learner-focused approach, for example, is driven primarily by particular beliefs about mathematics and how students learn it, while the two content-focused approaches describe learning activity first in light of the structure and scope of mathematics to be learned.

A Structure for the Study of Teachers' Knowledge, Skills, and Dispositions

The purpose of describing the approaches is to form a basis for identifying essential knowledge, skills, and dispositions for effective mathematics teaching. In the area of teacher knowledge, our categories were influenced by the work of Shulman (1986) and Smith (1980). We also drew from other work on the role of subject matter knowledge in teaching (e.g., Smith, 1969; Dewey, 1916; Hawkins, 1973).

Domains of subject matter knowledge. We divide the domain of subject matter knowledge into mathematics and pedagogical mathematics. By mathematics knowledge we mean: (a) knowledge of the content of school curriculum; (b) knowledge of mathematics that forms the foundation of school mathematics; and (c) knowledge about mathematics as a discipline and of other selected topics in higher mathematics. The second area, pedagogical mathematics knowledge, is suggested by Shulman in his discussion of "pedagogical content knowledge," the understandings in a field that are essential for teachers but may not be important for mathematicians or engineers. Pedagogical mathematics involves the ability to see and describe mathematics in ways that support learning. For example, providing effective explanations requires a unique understanding of the language and symbols of mathematics as well as the ability to represent and model mathematical ideas and concepts using familiar objects and situations.

Our selection of essential subject matter knowledge was logically derived from the approaches to mathematics and guided by the recommendations of various professional organizations (e.g., The National Council of Teachers of Mathematics' Commission of Post-war Plans, 1945; The National Council of Teachers of Mathematics' Commission on the Education of Teachers, 1981; The Mathematics Association of America's Committee on Undergraduate Programs in Mathematics, 1983).

Other knowledge and skill domains. Another knowledge area, "curricular knowledge," is described by both Smith (1980) and Shulman (1986) as a category of important information and understandings for teachers, involving knowledge about resources, materials, and tools of teaching in a discipline. This domain would include mathematical items and strategies, as well as materials and resources to support applications of mathematics in other fields of study. Knowledge in this area grows out of teachers' knowledge of mathematics and of other disciplines. For example, a teacher who is familiar with Islamic Art might know that prints of the tiled floors can be used to teach the geometric concepts of symmetry and tesselation.

Beyond knowledge related to academic disciplines, we also consider what we call "pedagogical knowledge and skill," referring to generic understandings and abilities that are important for effective teaching of mathematics as well as other school subjects. For instance, teachers need to understand ways to stimulate student interest and encourage their participation. They must also be able to organize and manage instructional materials.

The final domain, "dispositions," recognizes that while a lack of knowledge and skill may limit what teachers do, knowledge or skill alone will not necessarily lead to effective teaching (Katz & Raths, 1985; Wilson, 1975). This category is being defined to include teachers' orientations, commitments, and qualities of the mind that incline them to act in certain ways in the classroom (Feiman-Nemser, 1986). For example, a teacher using the learner-focused approach to teaching mathematics needs to see mathematics as a creative discipline and believe that learning is supported when students are actively involved in explorations of ideas.

Approach One: Learner Focused Approach

In this approach, the learner is the focus of classroom activity. Advocates of this approach typically hold a constructivist view of mathematics learning as described by Confrey (1985) and Cobb and Steffe (1983). From this perspective, learning is a process of constructing understandings of mathematics using the disciplinary methods of inquiry. Ultimately, it is the student's responsibility to evaluate the adequacy of his/her constructions (Confrey, 1985). However, the teacher aids the student by questioning, challenging, and offering experiences that reveal the inadequacy of inappropriate conceptions. When a student "knows" something, the student's construction of the idea is reasonably consistent with the actual meaning of that idea in the discipline. The student's ability to validate conjectures and respond appropriately to challenges to the constructions are accepted as evidence of knowing. Those who see mathematics as a method of inquiry or a way of thinking, rather than a mere record of knowledge, value this approach because it provides an opportunity for the student to "do" mathematics. They contend that the approach supports the development of problem solving skills and leads to a study of content that truly suits the needs and interests of the student.

The teacher's role in using this approach is to stimulate and facilitate student learning. Dienes (1972) explains that the teacher must be less authoritarian and that the exploratory situation, not the teacher, should provide answers to students. The teacher stimulates thought by posing problems, designing experiences, and asking questions which initiate and direct student exploration. The teacher facilitates student learning by responding to what students do and say--listening, probing, accepting, restating, encouraging, and providing counterexamples.

The pure form of this approach is somewhat incompatible with schools as they are organized, because of its focus on the individual learner, not groups of learners, and its lack of defined curriculum. The curriculum and the content of instruction is driven by the individual learner's interests, needs, and thinking. The use of written curricula that define the scope and sequence of content for classroom lessons is therefore inconsistent with this view of learning. The teacher using this approach does not feel compelled to follow a fixed schedule of content to be "covered," and is likely to pursue student tangents or areas of intense interest that grow out of initial investigations. Curriculum approaches discussed in the literature on the education of the "gifted" (e.g., Torrance, 1979; Renzulli, 1977) represent the only common school practices that are completely consistent with this approach to teaching.

Nonetheless, the underlying orientation, focusing on the learner's construction of knowledge with the teacher serving as facilitator, offers a plausible approach to classroom instruction. In the classroom formulation, lessons are organized around activities to be completed by individuals or by small groups of students working cooperatively. The teacher poses provocative problems for student investigation, providing only enough direction and explanation to communicate the task and stimulate interest. The use of cooperative small-group configurations to organize the classroom is seen as a means to maximize student engagement in the critical mathematical processes.

In this approach to teaching, the learner is an active participant in the exploration of ideas. Alternative methods for exploring problems are valued. Students talk a lot, to one another and to the teacher. The small group setting provides opportunities for students to become active critics of other students' approaches to problems, and also thoughtful defenders of their own approach.

Essential Knowledge, Skills, and Dispositions

Mathematics. Those who use this approach must be aware of the network of logical relationships that exist within a system of mathematics. If content decisions are to be driven by the learner's interest and successes, the teacher must recognize the variety of ways that content can be meaningfully structured using these logical relationships. For example, although a teacher's understanding of decimal numeration might be based on an understanding of the relationship between decimal and common fractions, the teacher must also be prepared to respond to the learner who "stumbles on to" decimals as an extension of the place value concept. This may require a discussion of decimals before the student has studied common fractions, and the teacher must know how this can be done.

In addition to understanding the range of mathematical relationships and structures that can guide the sequence of student experience, this teacher would benefit from an understanding of the history of mathematics, knowledge of some of the classical problems, and of content that may be only peripherally related to the school curriculum content. Such knowledge enables the teacher to construct worthwhile problems. An appreciation of the role of proof in mathematics helps the teacher judge the quality of student work. This is especially important because students' explorations may lead to curiosity about areas not typically considered in school mathematics. For example, a television program or a discussion in a science class may lead the student to approach a given task in a unique way or to ask about mathematics that does not appear in the regular mathematics textbook. In such cases the knowledge of mathematics outside school curriculum content prepares the teacher to respond to the student's question, evaluate the adequacy of the student's unconventional approach to the classroom task, or propose an investigation.

Finally, because the learner-focused approach is a method involving extensive problem solving as the primary mode of learning, the teacher must have ability to deal with mathematical problem solving tasks. The teacher must be able to solve problems using a variety of methods and approaches and know which approaches are within the scope of students' understanding.

Pedagogical mathematics. In using this approach, teachers must often be able to put aside their advanced knowledge of mathematics in order to guide the exploration of problems or ideas using what may appear to be inefficient or circuitous approaches. They must also have insight into students' naive approaches and be able to discriminate fruitful approaches to problems from those based on misconceptions or those having inherent limitations.

Knowledge of pedagogical mathematics also involves a unique understanding of the language of mathematics, its symbols and terminology. Teachers must be able to listen to students and watch them to detect seeds of important ideas or symptoms of misunderstanding. Their knowledge of mathematics terminology is used in two ways. In some cases the teacher might use the technical language to offer a restatement of a conjecture made by students. This would test and extend the depth of student comprehension. In other cases, the knowledge might be used only in reflective evaluation of students' discussions and descriptions of concepts and ideas. The key, in both cases, is to insure that students' formulations and their written and oral expressions of ideas are consistent with the full meaning of the precise and compact language of formal mathematics. If students do not have complete understanding of the complexity of a concept they are exploring, the teacher must decide whether or not the naive understandings have the potential to lead to future misconceptions. If so, the teacher must challenge students' discussions of the topic or provide further experiences to refine understandings; if not, further exploration might be reserved for another time.

The ability to model mathematical ideas using a variety of representations is also important for teachers. Dienes (1972) challenges teachers to use their understanding of modeling strategies to create and supervise a classroom that is a "mathematical environment." Kline's (1970, 1973) many criticisms of mathematics education are based on his argument that teachers need to be liberated from the focus on the logic of mathematics and learn to develop students' intuition through experience with the familiar. Hawkins (1973) refers to this as the ability to see mathematics through the eyes of students. These suggestions imply that teachers must know which materials, objects, and situations in the students' experience can form a bridge between the students' intuitive understandings and the formal mathematics to be learned. Teachers must also have an understanding of how these items can be used to stimulate curiosity and arouse interest in the study of mathematics.

At the secondary school level, teachers might make use of more formal modeling strategies. For example, an algebra teacher should be able to concretely illustrate topics that are studied with geometric models in much the same way that elementary teachers use concrete materials to illustrate the fraction or place value concepts. It is important for teachers to be able to stimulate students' grasp of the breadth of the idea by providing a number of different models for a single mathematical idea. At the same time, teachers should be able to use the particular models students find useful as a basis for exploring a wide range of concepts and ideas.

Curricular knowledge. Because the learner-focused approach does not lend itself to the use of a rigidly structured curriculum, teachers using this approach must have a personal storehouse of curricular knowledge to enable them to construct a curriculum for the particular classroom or student. The teacher must therefore know a great deal about existing curricula as well as know mathematics. For example, modern programs in mathematics are typically structured using a "spiral" approach, i.e., topics are revisited within and across grades to extend the students' depth of understanding. However, some curriculum resources are organized topically (e.g., addition), while others (e.g., some computer software) are structured according to a particular hierarchy of perceived prerequisites.

A teacher who does not follow a published program closely must understand the structure and content of those programs to insure that students are not misled or short changed by selective use of materials and activities. The teacher should understand what can and should be done with a certain age group or in a particular high school course. Use of a learner-focused approach also implies the need for knowledge of the content of other school subjects because this teacher would want to offer integrated studies providing opportunities for students to explore mathematical ideas in the context of other school work.

The teacher's curricular knowledge should also include an awareness of the various materials and resources that are available and how they might be used to support student learning. Judgments about which instructional materials to use should be guided by an understanding of the particular topics that are difficult or easy for students, the likely misconceptions, and the types of errors that are typically made. Changes in our world and in schooling also suggest that the learner-focused teacher should know about uses of technology in the classroom. Many of the materials being developed to support this approach to teaching use calculators or computers as the tool for student exploration. The teacher must understand the use of these newly developed materials in order to judge their worth in light of the students in the classroom, and to guide the students' work with the materials.

Pedagogical knowledge and skill. Because this teaching approach results from studies of learning rather than teaching, the literature provides little discussion about the classroom teaching skills essential for use of the approach, a deficiency noted by Silver (1986). Descriptions of essential teaching skills must therefore be inferred from discussions about the types of learning experiences that should be offered.

The classroom teaching skills used in this approach are different from those typically described in discussions of effective classroom management. When the learner-focused approach is used, much of the class time is spent having small groups of students work on tasks while the teacher circulates among the various groups. Thus the teacher must be able to skillfully design tasks and present them to students, stimulating interest and providing a clear explanation about what is expected. Effective use of the learner-focused approach also requires that the teacher be aware of group dynamics. Knowledge of both cooperative and competitive learning settings would also be helpful. The teacher must be able to apply those understandings to form instructional groups and to motivate learners to fully participate in group activities.

Since much of the "instruction" occurs during conversational interactions between the teacher and student, teachers must be able to conduct such sessions. This typically requires understanding of different questioning techniques and the deliberate use of convergent or divergent questions in specific situations. Teachers should be able to use strategies employed in clinical interviews where a range of questions related to a concept are posed to reveal the student's construction of that concept. Also teachers must have some strategy for keeping track of student progress across time.

Teachers using this approach need considerable knowledge about learning and learners. Cognitive psychology is the basis of the view of learning implied by the model and teachers must have an understanding of how to apply the principles of cognitive psychology to classroom practice. For example, teachers need an understanding of theories of cognitive development and recognize the implications of those theories for different students and for the various levels of schooling.

Finally, since much of the classroom activity is not under the direct supervision of the teacher, an awareness of theories of motivation and skill at planning both intrinsic and extrinsic rewards in the classroom environment are important. This is only one dimension of the essential skill of maintaining a productive and orderly (but not necessarily quiet) classroom. The learner focused approach is based on the assumption that a group of students who are presented with interesting and challenging tasks that are within the scope of their ability will be naturally engaged in the tasks without receiving special attention from the teacher Therefore, classroom management is not a matter considered in discussions of this teaching approach. The even flow of activity is assumed to exist. Of course, teachers are apt to need such ability even though it is not the focus of concern when this teaching approach is discussed.

Dispositions. Perhaps because, of all the teaching approaches, this one entails the most personal interaction between the learner, the teacher, and the content, the learner-focused approach may imply the broadest set of personal qualities (dispositions) for teachers. Those whose approach to teaching focuses on the learner must care about and respect students' ways of making sense of the content to be learned and trust that students can construct appropriate ideas. If they are to pursue tangents introduced by students' comments and questions, teachers must be fascinated with how students think and be intrigued with alternative ideas. They must be flexible in their thinking and have a tolerance for "mucking around" in ways that are not always efficient or orderly.

Those who use a learner-focused approach are apt to have a particular view of mathematics. They see the subject as more than a record of knowledge and skills. Mathematics is seen as the creation or "construction" of the learner and one important outcome of instruction is learning to investigate and construct ideas. Effective teachers, therefore, enjoy learning and "doing" mathematics themselves. They must care about understanding--their own and their pupils'-- and must see their role as one of facilitating and stimulating, not centrally directing learning or providing information.

The teachers must be fascinated with, and thus always be looking for, connections among mathematical ideas as well as connections between mathematics and the world. To improve their practice, teachers should care about studying mathematics themselves and be constantly seeking mathematical problems that are likely to provoke fruitful student exploration.

Approach Two: Content Focus with Emphasis on Understanding

This approach makes mathematical content the focus of classroom activity, but it emphasizes helping students develop an understanding of ideas and processes. The assumption is that the record of mathematical knowledge (including concepts, facts, rules, and ways of thinking) is appropriate for determining the curriculum, but that meaningful learning depends on students constructing their own understandings of mathematical ideas. Advocates of this approach recognize a distinction between what Skemp (1978) calls "relational understanding," knowing why things work as they do, and "instrumental understanding," knowing how to do something.

Learning, from the perspective of this approach, means that students acquire what is referred to as "conceptual knowledge," "conceptual understanding," or "meaningful understanding." These terms refer to both types of understanding as described by Skemp. When teachers use this approach, the ability to get correct answers, use algorithms, and recite definitions that may have been learned by rote, is not adequate evidence of "knowing" mathematics. Research involving clinical interviews (Erlwanger, 1975) or the analysis of semantic concept maps (Leinhardt and Smith, 1985; Leinhardt, 1985) supports the argument that many people who perform adequately on routine mathematical tasks may still hold significant misconceptions about the mathematical content. Thus, the criteria for evaluating learning in this approach are much like the criteria in the learner-focused approach.

The difference between this approach and the learner-focused approach is that in this approach the teacher has some view of scope and sequence of content to be learned, and the priorities for learning are derived from a view of the structure of the subject matter. There are differences of opinion, however, about how the content should be structured. For example, Noddings (1985) argues for an epistemological structure that differentiates among naive, formal, and metamathematical ways of knowing mathematics, while Cooney, Davis, & Henderson (1975) suggest domains based on types of knowledge, i.e., concepts, generalizations, singular statements, prescriptions, value judgments, and skills. Many mathematicians and school textbook authors tend to argue for a structure emerging foremost from the organization and logic of the discipline.

Each of these views recognizes and incorporates the ideas of the others in their conceptions of "good" teaching, but their suggestions about teaching vary somewhat because they are influenced heavily by one or the other opinion about how the curriculum should be organized.

Unlike the learner-focused approach, therefore, student ideas and interests are not primary in determining the curriculum. Teachers might be somewhat flexible about the sequence in which topics are presented, but they would not be inclined to pursue a tangent brought up by a pupil if it was not within the scope of knowledge they intended to explore with the class. Like the learner-focused teacher, however, the teacher using this approach would respect students' ideas and ways of thinking and would encourage the use of novel methods or inventions, believing that pupils' active construction of mathematics will lead to meaningful understanding of concepts.

The content-focused approach emphasizing conceptual understanding is unique from the other three because of the dual influences of content and learner. On one hand, content is focal, but on the other, understanding is viewed as constructed by the individual. It puts many demands on the teacher, and presents her with some tricky dilemmas.

First, the underlying principle is that lessons should be offered in ways that fit the content to be learned and help the particular learners develop an understanding of the content. Classroom instruction, and the roles of student and teacher, therefore, will vary considerably from lesson to lesson, requiring a wide range of pedagogical skills. On some occasions, the teacher may present information using a direct, expository approach, and on others, an inductive theory-building approach may be used. The teacher may present, model, or explain ideas, pose problems for students to investigate, ask questions, and respond to what students say and do, or at times merely present opportunities to practice what has been studied. The decision about which strategy to use is based on the type of content to be taught and the kind of support students need to attain understanding. A framework for guiding this eclectic approach to teaching is offered to secondary teachers by Cooney, Davis, & Henderson (1975) and to elementary teachers by McKillip, Cooney, Davis, & Wilson (1978).

One significant tension for the teacher involves pacing and allocation of time. On the one hand, teachers are driven by content considerations, which imply the need to cover the range of topics that comprise the school curriculum. On the other hand, the concern for developing meaningful understanding implies a willingness to explore topics in sufficient depth and in alternative ways. The teacher who takes this approach must cope with these competing concerns. In teaching a unit on multiplication, for example, it may seem fruitful to spend several weeks exploring the concept and its connections to the real world (see Lampert, 1986, for a description of such an exploration), yet the fourth grade curriculum includes a vast array of other topics which also demand the class' attention. In the learner-focused approach, the flexible view of content, minimizes this tension for teachers.

Another dilemma centers on the need to work with groups of students in an approach which takes individual understanding seriously. While this tension is not unique to a particular teaching approach, it is particularly acute here. Students make sense in different ways. The teacher must respect and work with individual students while providing opportunities for everyone to learn. Time is a problem. Acknowledging multiple ways of thinking can prove inefficient, and may even confuse students. Yet, ignoring alternative ideas can limit student learning.

Essential Knowledge, Skills and Dispositions

Mathematics. To teach in this approach, teachers need a solid foundation in mathematics. While textbooks and other curriculum materials provide sources of content and ways to approach it, teachers must also tailor and adapt instruction to help build bridges between pupils' understandings and disciplinary knowledge. If they are to strive for their pupils' conceptual understanding, then they themselves must understand mathematics conceptually.

In addition to knowledge of the discipline, knowledge about mathematics is also important for these teachers. As in the case of the learner-focused teacher, knowledge of the logical relationships within the system of mathematics is important. Although this teacher will be following a pre-planned sequence of content, an understanding of how concepts are related, intellectually equips the teacher to help students make connections among concepts. Like the learner-focused teacher, this teacher will benefit from understanding the evolution and growth of mathematical knowledge in the disciplinary community. For instance, a teacher who knows about the development of numeration systems has a wider base from which to evaluate curriculum materials or construct worthwhile problems for student investigation. Understanding the role of proof in mathematics can contribute to a teacher's perspective on how and when to ask pupils to justify their answers.

Beyond such disciplinary or foundational knowledge, since teachers teach the content of the school curriculum, they must also understand the subject matter that comprises school mathematics (Smith, 1969). For example, while teachers' visions of mathematics can be enriched by studying calculus, they must also have a solid conceptual understanding of fractions. In the past few years, the school curriculum has been expanded to include many topics that today's teachers did not study themselves in elementary or secondary school (e.g. probability, non-traditional problem solving). It is especially important for teachers to have an understanding of these areas or they may be inclined not to teach the content.

Pedagogical mathematics. The teacher in this approach is more than a facilitator; he/she is also mathematically and pedagogically active. The teacher must be able to represent ideas in a variety of ways and use alternative models and materials to illustrate concepts. But because the teacher has greater control over the students' explorations, this teacher may not need as wide a repertoire as the learner-focused teacher does. Yet, like the learner-focused teacher, this teacher must be able to listen to and watch children, and recognize the seeds of important mathematical ideas in the explorations of young learners (Dewey, 1916; Hawkins, 1973).

Once again, understanding of the language and symbols of mathematics is important, both to insure appropriate communication of ideas in the classroom and to support the evaluation of student work. Teachers should also be able to generate a range of appropriate examples or situations that fit a particular concept, and solve problems using strategies that students can comprehend, even if those strategies do not involve the most direct mathematical approach. Teachers must know some of the common ways children think about certain mathematical ideas (e.g., believing that squares are not rectangles) so that they can effectively challenge their pupils to stretch and grow.

Curricular knowledge. While the content drives decisions about what to teach, teachers shape their instruction in response to pupils' thinking. This implies the need for special types of curricular knowledge. Because the approach emphasizes helping students develop conceptual understanding, teachers must have a perspective on how mathematical topics spiral throughout the school curriculum. An eighth-grade teacher who teaches a unit on probability, for instance, should know about students' previous school experiences with probability concepts, as well as what they will explore in high school. An understanding of the curriculum spiral will guide decisions about where to begin the exploration of a concept with a particular grade-level group, and about whether or not the students' level of understanding is sufficient at a particular time.

The focus on student understanding also suggests that it may be necessary to supplement the basic class materials (e.g. the textbook) from time to time. Teachers may need to pull ideas from several textbooks or programs in the course of helping a group of students explore a particular topic. Thus, teachers must know how to make sense of and compare alternative curriculum materials. They must also recognize the topics that will be difficult for students and the approaches that might be used to overcome student difficulties.

Like the learner focused teacher, this teacher must be aware of modern curriculum resources (e.g., calculators and computers) and know how they might be used in the classroom. Since the emphasis is on helping students construct mathematical understandings within a set curricular framework, the teacher must be able to adapt curricular materials and activities to work with particular students, a skill that is often taken for granted (Ball & Feiman-Nemser, 1986). Teachers must also be able to create their own materials, when suitable ones cannot be found.

<u>Pedagogical knowledge and skill</u>. Clearly, this approach requires considerable pedagogical skill. First, since the approach calls for a variety of strategies to develop topics depending on the content to be learned, the teacher must have a broad repertoire of instructional strategies. Skill in presenting clear explanations and demonstrations is required when an expository mode is used. Skillful questioning strategies and the ability to spontaneously

generate examples are needed on occasions when the classroom activity is inquiry oriented. In addition, the teacher must be able to select strategies that are appropriate for teaching a particular topic or lesson.

Interactively, a teacher must be able to engage pupils in the tasks for the day. Because classroom activity will vary considerably, the teacher must be able to manage a classroom where students experience diversity of activity. During lessons where discussions are conducted, managing these discussions is a crucial skill--getting students to listen to one another and respect each others' ways of thinking. A teacher must be able to manage a loose classroom organization skillfully, and also be able to motivate students to be on-task when the class session is more traditional and routine. Even the physical arrangement of the classroom is important; the teacher needs to be able to arrange the space in a way that makes it possible to have group discussions at some times, and at others have small groups working, or space for undisturbed independent activity. This approach also requires considerable skill in assessing what students understand. The teacher must be able to come up with questions and tasks that afford a window on how their students are thinking about the content. Like the learner-focused teacher, this teacher must also be able to keep track of student learning across time and must also be able to organize and manage the curriculum materials and manipulatives that are to be used in the classroom.

Dispositions. Teachers who take this approach must have a tolerance for the inherent tension between "covering" the curriculum on one hand and helping students construct meaningful understandings on the other. Like teachers who embrace a learner-focused approach, they must care seriously about students' thinking and be fascinated with novel ideas. However, because of the content focus, these teachers must also care about providing students the opportunity to study ideas, skills, and concepts that are part of the mathematics studied as a school subject. These teachers must possess what may seem like contradictory qualities: organization and tolerance for inefficiency, orientation to "correct" knowledge and a respect for alternative formulations, a desire to lead at times and to step back and respond at others. To improve their teaching, these teachers should be constantly seeking new strategies for presenting mathematics in ways that promote student understanding (e.g., by searching for curriculum materials or by attending workshops or taking mathematics methods courses).

These teachers and learner-focused teachers may have similar conceptions of the discipline of mathematics. These teachers' conception of mathematics curriculum, however, will be influenced by one of the theorists who propose structural views of mathematics. For the most part, these teachers will value formal mathematics. They will want students to learn the record of knowledge in mathematics and will want them to understand relationships within the system of mathematics.

Approach Three: Content Focus with Emphasis on Performance

Like the content-focused approach described above, this approach to teaching takes mathematics content as its starting point. However, significant differences in beliefs about mathematics, about schooling, and about learning make this approach quite different than either of the other two. First of all, mathematics is viewed from a psychological perspective, rather than a disciplinary orientation. Organized in terms of a learning hierarchy of skills and concepts, the content is presented sequentially to students. Classroom activity is focused on helping students master the content of the curriculum, but in this case the emphasis is on knowing how to complete the exercises and problems in textbooks and on tests. Such performance is accepted as evidence of learning.

The difference between the understanding-focus and the performance focus in their conceptions of appropriate content for school mathematics is exemplified in Gagne's (1983b) rebuttal to Steffe & Blake's (1983) criticism of his views:

> The apparent view of Steffe and Blake that understanding involves some aspects of the "structure of mathematics" is what I would be inclined to question. I realize that this is an extremely common view among mathematics educators ....It will perhaps be apparent that I believe much mathematics instruction in grades K-12 should have the aim of teaching students how to use mathematics in their daily lives and in their continued pursuit of further education. I tend to think of mathematics instruction in these grades as having no peculiar value for someone who wishes to become a mathematician. Thus I consider that whatever "structure of mathematics" may be learned in these grades is unlikely to contribute significantly to the knowledge of that student who later decides to study mathematics as a scholarly discipline.

Many mathematics educators and educational psychologists favor this approach to teaching mathematics (e.g. Bloom, 1986; Gagne, 1983a, 1983b; Saxon, 1984; Scandura, 1972, 1973; Silbert, Carnine, & Stein, 1981). Some of their central premises include:

1. Rules are the basic building blocks of all mathematical knowledge and all mathematical behavior is rule-governed.
2. Knowledge of mathematics is being able to get answers and do problems using the rules that have been learned.
3. Computational procedures should be "automatized".
4. It is not necessary to understand the source or reason for student errors, further instruction on the correct way to do things will result in appropriate learning.
5. In school, knowing mathematics means being able to demonstrate mastery of the skills described by instructional objectives.

In this content-focused approach emphasizing performance, the selection of content for instruction begins with a hierarchy of prerequisite skills, and an assessment of student performance in light of that hierarchy. The teacher presents material in an expository style, demonstrating, explaining, and defining concepts and skills. The teacher's questions are more likely to be convergent, seeking answers (e.g., "What are the factors of 56?"), than in the earlier approaches. Students listen, participate in didactic interactions (e.g. responding to teacher questions) and do exercises or problems using procedures that have been modeled by the teacher or text. Advocates of the first two approaches would describe this as a passive student role because the student is not actively involved in exploring the content. Nonetheless, a considerable amount of student activity would be witnessed in observing a lesson based on this approach. For instance, students might work homework exercises at the board or might work with flash cards in small groups.

In the classroom, instruction may be directed to the whole group or to small groups (depending on whether or not the large group is homogeneous with respect to mathematics achievement). Many self-paced instructional programs also follow this approach, using written curricular materials (e.g. workbooks) to present information and a system of tests to monitor student mastery. Another important dimension of this teaching approach is that considerable time is allocated for students to practice the rules, procedures, or skills that are taught. In the case of computational skills, advocates of this approach suggest that repeated practice (often called drill) should be offered so that the skill will become "automatized."

Essential Knowledge, Skills, and Dispositions

Mathematics. While many people advocate that all teachers should have a solid grounding in their disciplines, teachers using this approach mainly need to know the mathematics of the school curriculum. For example, they should be able to compute accurately, and know the procedures entailed in common algorithms (e.g., long division, carrying). Because of the focus on helping students master curricular objectives, a broader disciplinary base is less directly crucial than it is in the two previous approaches, although professional groups (e.g., The National Council of Teachers of Mathematics and The Mathematics Association of America) advocate the need for foundational knowledge of mathematics for all teachers.

Pedagogical mathematics. Pedagogical mathematics knowledge for this approach is distinctively different. Although this approach does not place emphasis on understanding as described by Skemp (1978) and others, teachers using this approach must be able to explain the mathematical rules and procedures to students and illustrate them with examples. Teachers must understand the hierarchical structure of mathematics, the ways in which particular skills and concepts are prerequisite to others. This knowledge is important in helping teachers appraise student difficulties, make decisions about student progress, and use and adapt curricular materials. Knowing the particular topics that students find difficult (e.g., division with zero in the quotient) is important to facilitate effective planning. Anticipation of student difficulty should inform decisions about pacing and the amount of practice that is necessary. Teachers should also be aware of certain strategies (e.g., mnemonics) that might help students master procedures.
As in the case of the other approaches, teachers must have facility with the language and symbols of mathematics and be able to communicate their meaning to students.

Curricular knowledge. Since this approach usually relies on curricular materials, either textbooks or kits of instructional activities, teachers' curricular knowledge is critical to successful teaching. They need to be able to understand the intent and structure of these materials in light of the mathematics to be learned and the beliefs they hold about learning to guide the sequence and pace of instruction. This knowledge serves as a basis to enable teachers to select and adapt materials to insure pupils' mastery of mathematical procedures. They also need to understand assessment tools or management plans that are associated with the materials used, to inform their decisions about what should be taught, when, and to whom.

Pedagogical knowledge and skill. Teaching for performance requires a range of pedagogical skills. Many of these skills are related to the need for remediation experiences for individuals. This need is the natural outgrowth of a performance-focused program. The teacher must be able to assess what students can do, determine what skills they need to practice, and at what level they need new instruction. Systematic record-keeping is important to monitor pupil progress. The teacher must be able to motivate students to complete assignments and participate in practice activities (e.g., games to drill multiplication facts). Knowing how to support students' efforts to master complicated procedures, such as borrowing or factoring polynomials is also important. Since the teacher in this approach is committed to covering content, they must be able to make skillful judgments about how to proceed efficiently through the curriculum, making good use of time across the school year and structuring class time in a way that helps students increase their mastery of the material.

Skills in classroom management are essential. Materials and space must be organized to allow students to engage in the instructional tasks with a minimum of confusion. The teacher must monitor and manage student activity to insure that their efforts are productive.

Dispositions. Teachers who take a content focus and emphasize student performance must care about "covering" the curriculum. Committed to taking a directive, authoritative stance, they must feel responsible to provide access to, and ensure mastery of, school mathematics. This implies that they must appreciate clear organization of the content and direct, unambiguous explanations and procedures. They should be committed to monitoring student progress by regularly checking what students know, remember, and can do. Teachers should have a drive to provide closure and enjoy "systematizing" mathematical knowledge (e.g. teaching long division as a series of steps). They have a particular interest and concern about their students' success in school. Viewing school mathematics as focused on performance, mathematics itself is primarily seen as a collection of structured and related rules and procedures to be mastered. To improve the quality of the instruction they offer, teachers should care about looking for new materials that are structured in ways that may make student progress more efficient.

Approach Four: Classroom-Focused

This approach focuses on the importance of well-structured and efficiently organized classroom activity. The approach is primarily characterized by its prescription of appropriate teacher behaviors that are derived from the findings of process-product research on teaching. The model does not address questions about the curriculum and the content of instruction; it assumes the existence of a school curriculum that becomes the determinant of instructional content. Particular views about how learning occurs or what it means to know are not asserted by this teaching approach, although a view of understanding from the perspective of the performance-focused approach is common.

When this approach to teaching is used, the teacher is an active instructor, clearly presenting material to the whole group and providing individual practice for students. Good teachers skillfully explain, assign tasks, monitor student work, provide feedback to students, and manage the classroom environment, preventing, or eliminating, disruptions that might interfere with the flow of planned activity. The student's role is to be a cooperative and attentive learner, following teacher directions, answering questions, and completing assigned tasks. A description of effective instruction from this perspective concentrates on the flow of activity and the types of events that transpire during lesson. The substance of teacher-student interactions are not specified. The assumption is that students learn best when classroom lessons are clearly structured and follow principles of effective instruction (e.g., maintaining high expectations, insuring a task-focused environment).

Perhaps because the classroom-focused approach does not treat issues related to teaching specific school subjects, it is often the basis for initiatives to assess teachers' competence using formal observational instruments (e.g., state teacher evaluations in South Carolina, Georgia, Texas). Madeline Hunter's popular method, emphasizing pedagogical skill and the structure of classroom lessons, also illustrates this generic classroom-focused approach to teaching subject matter. The Missouri Mathematics Program (Good, Grouws, & Ebmeir, 1983) might be seen as an example of this teaching approach applied to the context of mathematics instruction. This instructional system is focused on the classroom setting and the structure of lessons. It specifies five components of effective mathematics lessons: daily review, development, seatwork, homework, and weekly and monthly reviews of skills and concepts.

Unlike the work of Hunter and others who ignore content in the discussion of effective teaching, Good et al. (1983) treat subject matter issues in discussions of the development portion of a lesson. Good's (1986) recent discussion of effective mathematics teaching represents a departure from the pure classroom-focused approach and closely parallels the views of the understanding-focused approach. Actually, a discussion of teaching that begins with a classroom focus and then proceeds to discuss content, is likely to represent a hybrid version of the approaches described in this memo. Although their emphasis on lesson structure and classroom management characterizes such discussions as having a classroom-focused approach, the treatment of subject matter issues is bound to incorporate the views associated with one of the two content-focused approaches.

Essential Knowledge, Skills, and Dispositions

Mathematics. Because this teaching approach does not deal with the teaching of specific subject matter, it does not offer any perspective on the types of mathematics knowledge teachers need to teach that subject. This is not to say that subject matter knowledge is seen as irrelevant to good teaching in this approach. Rather, the focus of attention in characterizing good teaching from this perspective does not relate to the question of what knowledge of mathematics is essential. We can assume that the teacher would have to have command of the content of school curriculum.

Pedagogical mathematics. Once again, no specific recommendation can be discerned from the characteristics of good teaching using this approach. The need for knowledge of pedagogical mathematics would vary depending on the teacher's conception of mathematics curriculum (i.e., whether it most clearly resembles the first, second, or third approach discussed above).

Curricular knowledge. Knowledge of mathematics curriculum, however, is obviously essential for effective use of this teaching approach. Since the approach treats curriculum as a given, it assumes that teachers understand curriculum structure and how it relates to successful learning in the discipline. The teacher must be knowledgeable about trends, innovations, approaches, and principles that guide school mathematics curriculum. Knowledge in this domain includes such things as: understanding of the organization of curriculum materials to be used, awareness of common models and materials that are used to teach particular topics, recognition of changes in the content of school curriculum that are implied by changes in modern technology, and understanding of what can and should be accomplished within the study of mathematics with a certain age group or in a particular high school course.

The teacher's curricular knowledge should also include an awareness of the various materials and resources that are available and how they might be used in the classroom context implied by this approach. For example, if the school curriculum requires that students have experiences with computers, the teacher must be aware of software that would enable computer use to become part of the classroom routine. Students' on-task behavior is seen as a critical index of effective use of this approach. The teacher must be aware of the types of instructional materials and resources that result in high levels of student interest in classroom activities. Since specific parts of the classroom routine prescribed by this approach involve independent work for reinforcement or practice, the teacher probably needs more student tasks to support that type of student work than a teacher who does not follow this approach.

Pedagogical knowledge and skill. For the most part, the pedagogical skills that are the basis of this teaching approach match those described by Floden (1986). These skills relate to the planning of appropriate lessons, to the presentation of the lesson, and to the direction of classroom activity. To effectively use this approach to teaching, the teacher should be able to specify clear objectives for instruction and communicate these objectives to students. It is also important that the classroom activities be appropriate for students. The teacher should be able to select tasks such that the difficulty level is sufficient to stimulate student interest and sustain their participation in the learning activities. The ability to plan and manage time would clearly be important, insuring that adequate time is spent developing the content of the lessons while also allowing students enough time to practice and work independently. The focus of this teaching approach implies that managing student behavior in class is important. Teachers must be skillful at monitoring student behavior (knowing what the students are doing). They must also be able to engage inattentive students and deal with disruptive students in ways that do not interfere with the flow of classroom activity. Teachers can avoid student behavior problems by skillfully pacing classroom activity.

This teaching approach suggests that particular types of classroom interactions are productive and others are not. For example, the teacher must be aware of the types of questions that might be asked during lesson development and for assessment. The teacher must be skillful at using a range of questioning strategies (e.g., process-focused, product-focused, convergent, or divergent) and also know when a particular question type would be most useful.

Because student behavior is an important indicator of successful use of this approach, the teacher must be skillful at sustaining positive interpersonal relations during the class period. It is important that the teacher provide feedback to students to inform them of their success within the lesson (are answers correct or not). At the same time the teacher must do this so as to motivate, and not discourage, challenge and not embarrass or humiliate the students. A great deal of sensitivity is therefore needed for effective use of this teaching approach, despite the on-task business demeanor that is to be projected by the teacher.

The teacher's role is to present explanations of content clearly. Skillful communication is essential to success with the approach. The clarity of presentations, organization of information, and use of introductory and summary statements that help students focus on the learning goals are seen as important skills when this teaching approach is used.

Dispositions. In this approach, teachers must care about organizing the classroom environment to be task-focused and efficient. They should believe that an emphasis on well-structured lessons accompanied by regular practice will promote student learning. These teachers should care about improving their classroom teaching skills, such as explaining, providing feedback, and monitoring seatwork. They should be committed to monitoring themselves--the expectations they hold, the way they provide feedback, and how clearly they explain procedures and structure the learning environment. They should believe that they can improve their teaching by self-monitoring, by having someone else observe their behavior, and by attending workshops designed to enhance teacher effectiveness.

Summary

This discussion of teaching approaches offers examples of the knowledge, skills, and dispositions necessary to teach mathematics effectively. Some skills, understandings, or qualities are unique to a particular view of good mathematics teaching. Some are essential according to more than one conception. Some are simply important to conceptions of good teaching in general.

The purpose of this discussion is to identify aspects of good teaching that should be considered in studying how teachers learn to teach mathematics, and how teacher education programs influence teacher development. The task ahead is to determine which knowledge, skills, and dispositions should be expected of a beginning teacher and which are more likely to be outcomes of experience or inservice education and procedures for investigating these potential outcomes of teacher education must be developed.

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